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Self-Focusing Effect of an Electromagnetic Wave under Various Nonlinearities

Dr. Amrita
Associate Professor, Department of Physics, D. A. V. College, Muzaffarnagar, Uttar Pradesh, India

Corresponding Author: amritaiitd@yahoo.com

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ABSTRACT
A short pulse laser beam suffers self-pulse distortion due to combined effects of nonlinearity induced self-focusing and dispersion. Nonlinearity arises due to relativistic mass variation and ponderomotive force. As the beam propagates through plasma, beam width parameter (f) *decreases, so intensity of the beam increases. Self focusing of the beam takes place. As time passes, decrease in* (f) *is smaller and smaller, broadening of pulse takes place.*

Keywords: *electromagnetic, wave, nonlinearities, hydrogen*

I. INTRODUCTION

In plasmas, where relativistic mass and density nonlinearities predominate, one observes frequency downshift and leads to relativistic self-focusing of the laser beam [1-3]. Paraxial ray theory [4-5] of relativistic and ponderomotive self-focusing has been developed. Fedosejevs *et al* [6] have reported experiments on many gases including helium, hydrogen and nitrogen etc. They employed a $0.3TW, 250-fs$ laser pulse focused to an intensity of $3\times10^{17} W/cm.^2$ They observed relativistic self-focusing in high density hydrogen gas.

Frederick et al [07] have compared the importance of relativistic self-focusing in comparison with ponderomotive non-relativistic self-focusing at very high laser intensities. When laser intensity is high, ponderomotive self-focusing becomes dominant.

Borghesi *et al* [08] have observed relativistic self-channeling of intense laser beams both experimentally and in 3D Particle in cell [PIC] simulations. In tunnel-ionizing gases, the laser undergoes radial divergence, frequency upshift [9, 10], ring formation [11] due to nonlinear refraction.

Gibbon *et al* [[12] have experimentally studied relativistic self-focusing and self-channeling of an intense laser pulse in underdense plasma. Here, we study the self-focusing of intense short laser pulses in plasma dominated by relativistic mass nonlinearity along with change in electron density due to radial ponderomotive force. In section 2, we obtain coupled equations for amplitude and eikonal. In section 3, we solve the wave equation and obtain an analytical solution for the evolution of a laser pulse for general nonlinearity. In section 3(a), we discuss the case when laser intensity is moderate and nonlinearity is quadratic. In section 4, we discuss results.

II. COUPLED EQUATIONS FOR AMPLITUDE AND EIKONAL

Consider the propagation of a linearly polarized intense laser pulse in plasma in z direction. At $z=0$, the electric field of the laser is

$$\mathbf{E} = \hat{x}A(r,t)\exp(-i\omega t),$$

$$A_0^2 = A_{00}^2 \exp\left(-r^2/r_0^2\right)\exp\left(-t^2/\tau^2\right). \quad (1)$$

For $z>0$, we will note later that

$$A_0^2 = \frac{A_{00}^2 G(\tau,\xi)}{f^2}\exp\left(-r^2/r_0^2 f^2\right). \quad (2)$$

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Laser imparts drift motion to electrons. Momentum balance and energy equations are:

$$\frac{d\mathbf{p}}{dt} = -e\mathbf{E} - \frac{e}{c}\mathbf{v}\times\mathbf{B}. \tag{3}$$

$$mc^2\frac{d\gamma}{dt} = -e\mathbf{E}.\mathbf{v}. \tag{4}$$

If one treats the r, z, t variations of $\mathbf{A}$ to be small as compared to phase variations of $\mathbf{E}$, the x component of Eq.(3) gives

$$p_x = \frac{eE_x}{i\omega}. \tag{5}$$

The time average Lorentz factor is

$$\gamma = \left(1 + \frac{p_x p_x{}^*}{2m^2c^2}\right)^{1/2} = \left(1 + \frac{a^2}{2}\right)^{1/2}. \tag{6}$$

and the drift velocity of electrons due to laser is

$$\mathbf{v} = \frac{e\mathbf{E}}{mi\omega\gamma}, \qquad a^2 = \frac{e^2\,|A|^2}{m^2\omega^2c^2} = \frac{e^2A_{00}^2G\exp\left(-r^2/r_0^2f^2\right)}{m^2\omega^2c^2f^2}, \tag{7}$$

The laser also exerts a ponderomotive force on electrons,

$$\mathbf{F_p} = +e\nabla\phi_p, \tag{8}$$

In the equilibrium, the ponderomotive force on electrons balances the space charge force,

$$+e\nabla\phi + \nabla\phi_p = 0, \text{ leading to } \phi = -\phi_p = -\frac{mc^2}{e}\left(1 + \frac{a^2}{2}\right)^{1/2} \tag{9}$$

From Poisson's Equation, we obtain

$$n = n_0 - \frac{mc^2a_0^2G}{4\pi e^2}\left(-\frac{1}{r_0^2f^4} + \frac{1}{r_0^4f^6}\right)\exp\left(-r^2/r_0^2f^2\right). \tag{10}$$

The dielectric constant of the plasma can be written as

$$\begin{aligned}\epsilon &= 1 - \frac{4\pi ne^2}{m\omega^2\gamma} = 1 - \frac{\omega_{p0}^2}{\omega^2} + \left\{\frac{\omega_{p0}^2}{\omega^2}\left[\frac{1}{4f^2}\right] + \frac{c^2}{\omega^2}\left[\frac{-1}{r_0^2f^4} + \left(\frac{1}{4r_0^2f^6} - \frac{r^2}{4r_0^4f^8}\right)a_0^2G\exp\left(-r^2/r_0^2f^2\right) + \frac{r^2}{r_0^4f^6}\right]\right\} \\ &\times a_0^2G\exp\left(-r^2/r_0^2f^2\right) \\ &= \epsilon_0 + \phi\left(EE^*\right)\end{aligned} \tag{11}$$

The wave equation governing the propagation of high amplitude electromagnetic wave is

$$\frac{\partial^2E}{\partial z^2} + \nabla_\perp^2E - \frac{1}{c^2}\frac{\partial^2E}{\partial t^2} + \frac{\omega^2}{c^2}\left(\epsilon_0 + \phi - 1\right)E = 0. \tag{12}$$

Taking rapid phase variation as

$$\mathbf{E} = \hat{x}A(r,z,t)\exp\left[-i(\omega t - kz)\right], \text{ where } k = \frac{\omega}{c}\left(1 - \frac{\omega_{p0}^2}{\omega^2}\right)^{1/2} \tag{13}$$

Using above expressions in Equation (12), we get

$$\nabla_\perp^2A + 2ik\frac{\partial A}{\partial z} + \frac{2i\omega}{c^2}\frac{\partial A}{\partial t} + \frac{\partial^2A}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2A}{\partial t^2} + \frac{\omega^2}{c^2}\phi A = 0. \tag{14}$$

We have ignored $\partial^2 A/\partial z^2$ and $\partial^2 A/\partial t^2$ by using WKB approximation. Now introducing, $\xi = z$, and $\tau = t - \frac{z}{\mathbf{v_g}}$, where $\mathbf{v_g} = \frac{c^2 k}{\omega}$, Eq. (14) takes the form

If we take self-focusing length $z_f > r_0$, then we may neglect $\frac{\partial^2 A}{\partial \xi^2}$, so Eq.(14a) is

$$\nabla_\perp^2 A + 2ik\frac{\partial A}{\partial \xi} + \frac{\omega_{p0}^2}{\omega^2 c^2}\frac{\partial^2 A}{\partial \tau^2} - \frac{2}{\mathbf{v_g}}\frac{\partial^2 A}{\partial \xi \partial \tau} + \frac{\omega^2}{c^2}\phi A = 0. \tag{14a}$$

If we consider the case, where $L_p/z_f > \omega_{p0}^2/\omega^2$, with $L_p = ct_0 \in_0^{1/2}$, the length of the pulse and z_f, the characteristic length of self-focusing and neglect $\partial^2 A/\partial \tau^2$ term in Eq.(14a), then

$$\nabla_\perp^2 A + 2ik\frac{\partial A}{\partial \xi} - \frac{2}{\mathbf{v_g}}\frac{\partial^2 A}{\partial \xi \partial \tau} + \frac{\omega^2}{c^2}\phi A = 0. \tag{14b}$$

If we introduce an eikonal $A = A_0 \exp(ikS)$ and separate real and imaginary parts of Eq. (14b), we get

$$2\frac{\partial S}{\partial \xi} + \frac{2}{\mathbf{v_g}}\left(\frac{1}{k^2 A_0}\frac{\partial^2 A_0}{\partial \xi \partial \tau} - \frac{\partial S}{\partial \xi}\frac{\partial S}{\partial \tau}\right) + \left(\frac{\partial S}{\partial r}\right)^2 = \frac{\nabla_\perp^2 A_0}{k^2 A_0} + \frac{\omega^2}{k^2 c^2}\phi. \tag{15}$$

$$\frac{\partial A_0^2}{\partial \xi} - \frac{1}{\mathbf{v_g}}\left(\frac{\partial S}{\partial \xi}\frac{\partial^2 A_0}{\partial \tau} + \frac{\partial S}{\partial \tau}\frac{\partial^2 A_0}{\partial \xi} + \frac{\partial^2 S}{\partial \xi \partial \tau}.2A_0^2\right) + \nabla_\perp^2 S.A_0^2 + \frac{\partial S}{\partial r}\frac{\partial^2 A_0}{\partial r} = 0. \tag{16}$$

III. SELF-FOCUSING

We expand S, in the paraxial ray approximation $r^2 << r_0^2$ as

$S = S_0(\tau, \xi) + \beta(\tau, \xi)\frac{r^2}{2}$, where $\beta = \frac{1}{f}\frac{\partial f}{\partial \xi}$,

Hence, we may write S as $S = S_0(\tau, \xi) + \frac{1}{f}\frac{\partial f}{\partial \xi}\frac{r^2}{2}$. One obtains from eq.(16)

$$A_0^2 = G(\tau, \xi)F(\tau, \xi, r),\ F(\tau, \xi, r) = \frac{1}{f^2(\tau, \xi)}\exp\left[-\frac{r^2}{r_0^2 f^2(\tau, \xi)}\right]. \tag{17}$$

Substituting for A_0^2 and S in Eqs.(15) and (16), we get

$$\frac{\partial S_0}{\partial \xi} + \frac{1}{\mathbf{v_g}}\left[\left(\frac{1}{k^2 A_0}\frac{\partial^2 A_0}{\partial \xi \partial \tau}\right)|_{r=0} - \left(\frac{\partial S_0}{\partial \xi}\frac{\partial S_0}{\partial \tau}\right)\right] = \frac{\nabla_\perp^2 A_0}{k^2 A_0} + \frac{\omega^2}{k^2 c^2}\phi\,|_{r=0}\ . \tag{18}$$

$$\frac{\partial G}{\partial \xi} - G\frac{2}{f}\frac{\partial f}{\partial \xi} + 2\beta G - \frac{1}{F\mathbf{v_g}}\left(\frac{\partial S_0}{\partial \xi}\frac{\partial^2 A_0}{\partial \tau} + \frac{\partial^2 S_0}{\partial \xi \partial \tau}.2A_0^2\right) = 0. \tag{19}$$

In the first order approximation, we consider $\partial/\partial \tau = 0$, then

$$\frac{\partial S_0}{\partial \xi} = \frac{\omega^2}{2k^2c^2}\phi|_{r=0} - \frac{1}{k^2 r_0^2 f^2}, \quad \frac{\partial G}{\partial \xi} = 0.$$

The next order of Eq. (18) gives

$$\frac{\partial G}{\partial \xi} - \frac{1}{\mathbf{v_g}}\left[\left(\frac{\omega^2}{k^2c^2}\frac{\phi|_{r=0}}{2} - \frac{1}{k^2 r_0^2 f^2} + \frac{\partial \phi}{\partial A_0^2}A_0^2\right)\left(\frac{\partial G}{\partial \tau} + \frac{G}{F}\frac{\partial F}{\partial \tau}\right)_{r=0} + \frac{4G}{k^2 r_0^2 f^3}\frac{\partial f}{\partial \tau}\right] = 0. \quad (20)$$

Collecting second order terms of Eq. (19), we obtain

$$\frac{\partial^2 f}{\partial \xi^2} = \frac{1}{k^2 r_0^4 f^3} + \frac{\omega^2}{k^2c^2}\left\{\left[-\frac{\omega_{p0}^2}{\omega^2}\left[\frac{a_0^2 G}{4r_0^2 f^3}\right] + \frac{c^2}{\omega^2}\frac{2a_0^2 G}{r_0^4 f^5} - \frac{c^2}{\omega^2}\frac{3\left(a_0^2 G\right)^2}{4r_0^4 f^7}\right]\right\}.$$

$$= \frac{1}{k^2 r_0^4 f^3} - \frac{\omega^2}{k^2c^2}\frac{\partial \phi}{\partial A_0^2}|_{r=0}\ \frac{G}{f^3 r_0^2}.$$

(21)

3(a) Quadratic Nonlinearity

When laser intensity is moderate $a^2 < 1$, the nonlinear part of permittivity simplifies to $\phi = \in_2 A_0^2$. Now, if the variation of G is small over the self-focusing length $\frac{1}{G}\frac{\partial G}{\partial \xi} << \frac{1}{f}\frac{\partial f}{\partial \xi}$, then equation (22) can be solved analytically,

$$f^2 \approx 1 - \frac{\xi^2}{z_f^2}, \text{ where } \frac{1}{z_f^2} = \frac{\omega^2}{k^2c^2}\frac{G \in_2}{r_0^2}, \in_2 = \frac{\omega_{p0}^2}{\omega^2}, \quad (22)$$

Using this value in Eq. (21), we obtain

$$\frac{\partial G}{\partial \xi} - \frac{1}{\mathbf{v_g}}\left[\left(\frac{\omega^2}{k^2c^2}\frac{3}{2}\in_2\right)\frac{G}{\left(1 - \frac{\xi^2 G \in_2}{r_0^2}\frac{\omega^2}{k^2c^2}\right)^2}\left(\frac{\partial G}{\partial \tau}\right)\right] = 0. \quad (23)$$

We take initial variation of G as $G = \frac{a_0^2}{2}e^{-t'^2/\tau^2}$. We take the parameters as follows: $\frac{\omega_{p0}^2}{\omega^2} = 0.1$, $a_0^2 = 0.1$, $\frac{\omega r_0}{c} = 20$,

We solve equation (22) for these parameters. Variation of beam width parameter versus time is shown in (c. f. Fig. 1).

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IV. CONCLUSION & DISCUSSION

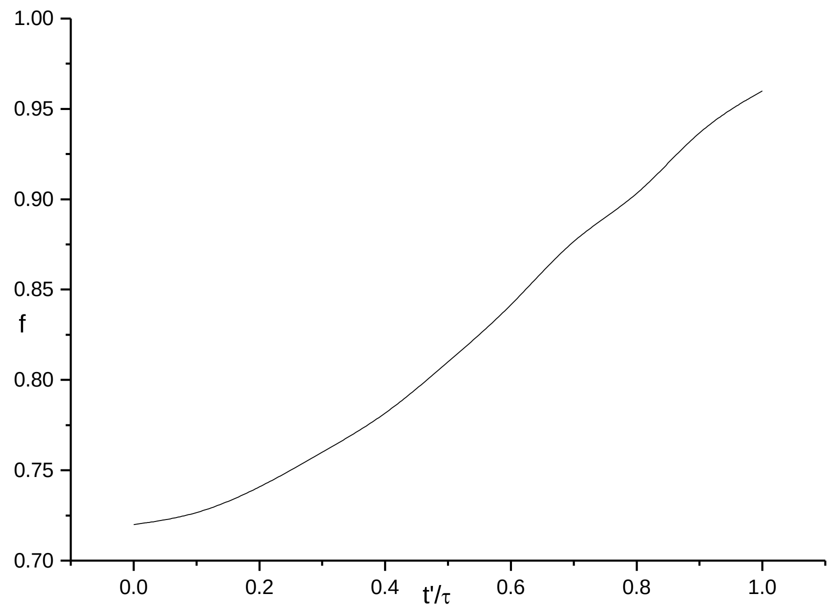

Figure 1: Variation of beam width parameter (f) versus relative time $\left(\frac{t'}{\tau}\right)$ for propagation distance $\frac{\xi}{R_d} = 0.4$

As the beam propagates through plasma, beam width parameter (f) decreases, so intensity of the beam increases. This is due to the convergence of the beam, the portion of the wavefront where intensity is maximum travels with minimum phase velocity while adjoining portions move with faster phase velocity, causing convergence of beam. Self focusing of the beam takes place. As time passes, decrease in (f) is smaller and smaller, broadening of pulse takes place. Hence due to ponderomotive and relativistic nonlinearities, broadening of the pulse takes place and intensity of the beam decreases due to temporal effect.

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